Long range neutrino forces in the cosmic relic neutrino background

F. Ferrer[a]**, J. A. Grifols**[a]**, and M. Nowakowski**[b]

[a]*Grup de Física Teòrica and Institut de Física d'Altes Energies*
Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Spain

[b]*Instituto de Física Teórica*
Universidade Estadual Paulista
Rua Pamplona 145, 01405-900 São Paulo, Brazil

Abstract

Neutrinos mediate long range forces among macroscopic bodies in vacuum. When the bodies are placed in the neutrino cosmic background, these forces are modified. Indeed, at distances long compared to the scale T^{-1}, the relic neutrinos completely screen off the 2-neutrino exchange force, whereas for small distances the interaction remains unaffected.

Dispersion potentials arising from double particle exchange have been systematically studied in a wide variety of physical contexts and with quite different scopes and purposes [1]. Indeed, the studies include pure QED phenomena such as Van der Waals interactions [2], two neutrino forces among macroscopic bodies [3], forces mediated by scalar particles [4, 5] found in recent completions of the standard model (e.g. superlight scalar partners of the gravitino), etc. In particular 2-neutrino exchange forces have been repeatedly scrutinized since first discussed by Feinberg and Sucher. An aspect that has been reanalysed in recent work [6] is the observation raised in [7] that the cosmic neutrino heat bath has an effect on long range neutrino interactions. In both these papers [7, 6] an approximate neutrino distribution function was used that simplified the calculations. The claim was that for small neutrino chemical potential, the background neutrinos can be considered nearly Boltzmann distributed and this fact, while only introduces a small distortion into the long range forces, makes the calculations much easier. But the actual phase-space distribution for relic cosmological neutrinos has a Fermi-Dirac shape. Indeed, any fermionic species in thermal equilibrium which at time t_D and temperature T_D of decoupling was highly relativistic followed an equilibrium distribution $n(\mathbf{p}, t_D) = [\exp{(E/T_D)} + 1]^{-1}$. After decoupling, the energy is red shifted by the expansion of the Universe, $E(t) = E(t_D)(R(t_D)/R(t))$, as the number density decreases like R^{-3}. As a result, the phase-space distribution at time t will keep the Fermi-Dirac form with the temperature $T(t) = T_D(R(t_D)/R(t))$. In the present paper we use the exact Fermi-Dirac neutrino distribution function with arbitrary chemical potential and observe that the long distance results are drastically modified even for small chemical potential in contrast to previous claims. We neglect the effect of a neutrino mass which for the phenomenologically suggested values would not affect the present results. We comment in passing that there has been in the recent literature [8] renewed interest in cosmic neutrino degeneracy which could make neutrino scattering a viable explanation for the Ultra-High-Energy Cosmic Ray events observed so far.

We shall adopt the notation in [7, 6] and write,

$$V(\mathbf{r}) = -\int \frac{d^3\mathbf{Q}}{(2\pi)^3} \exp(i\mathbf{Q} \cdot \mathbf{r}) \mathcal{T}(\mathbf{Q}) \tag{1}$$

where $\mathcal{T}(\mathbf{Q})$ is the nucleon-nucleon elastic scattering amplitude (Fig. 1) in the static limit, i.e. momentum transfer $Q \simeq (0, \mathbf{Q})$, where matter is supposed to be at rest in the microwave background radiation (MWBR) frame. It can be cast in the form

$$\mathcal{T}(Q) = -2iG_F^2(g_V, -2g_A\mathbf{S})^\mu(g_V', -2g_A'\mathbf{S}')^\nu I_{\mu\nu} \tag{2}$$

with

$$I_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} Tr[\gamma_\mu OiS_T(k)\gamma_\nu OiS_T(k-Q)] \tag{3}$$
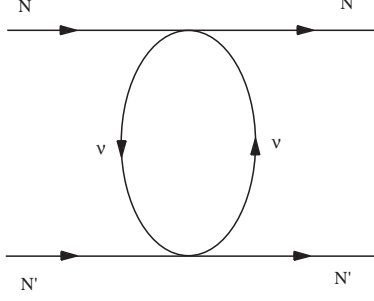
1

Figure 1: *Lowest order Feynman diagram for two neutrino exchange in the four fermion effective theory.*

and $\mathbf{S}, \mathbf{S}'$ are spin operators. The operator O is the left-handed projector $\frac{1}{2}(1 - \gamma_5)$ for Dirac neutrinos. The temperature dependent propagator S_T has the explicit form

$$S_T(k) = \not{k} \left[(k^2 + i\epsilon)^{-1} + 2\pi i \delta(k^2)(\theta(k^0)n_+ + \theta(-k^0)n_-) \right] \tag{4}$$

where n_+ and n_- are Fermi-Dirac distribution functions for particle and antiparticle, respectively. As discussed in [7], Fig. 1 evaluated with this propagator taken together with the usual Feynman rules is sufficient to calculate the potential. In equation (2), $g_{V,A}$ are composition-dependent weak vector and axial-vector couplings. We focus on the spin-independent potential, that is the $g_V g_V'$ component of Eq.(2). Physically this component in the potential arises because the helicity flip produced by single neutrino exchange is balanced by the exchange of the second neutrino and, as a consequence, a spin-independent interaction takes place that leads to a coherent effect over many particles in bulk matter. Use of the first piece in equation (3) gives the well known vacuum result

$$V_0(r) = \frac{G_F^2 g_V g_V'}{4\pi^3 r^5}. \tag{5}$$

In a neutrino background, a contribution to the long range force can arise because a neutrino in the thermal bath may be excited and de-excited back to its original state in the course of the double scattering process. This effect is described by the crossed terms contained in $I_{\mu\nu}$ that involve the thermal piece of one neutrino propagator along with the vacuum piece of the other neutrino propagator. This thermal piece of the tensor $I_{\mu\nu}$ can be written as

$$
\begin{aligned}
I_T^{\mu\nu} &= -\pi i \int \frac{d^4 k}{(2\pi)^4} \delta(k^2)[\theta(k^0)n_+ + \theta(-k^0)n_-] \\
&\quad \times \left[\frac{Tr\,[\gamma^\mu(\not{k} + \not{Q})\gamma^\nu \not{k}]}{(k+Q)^2 + i\epsilon} + \frac{Tr\,[\gamma^\mu \not{k} \gamma^\nu(\not{k} - \not{Q})]}{(k-Q)^2 + i\epsilon} \right].
\end{aligned} \tag{6}
$$

The temperature dependent potential

$$V_T(r) = \frac{iG_F^2 g_V g_V'}{\pi^2 r} \int_0^\infty dQ \, Q \, I_T^{00}(Q) \sin Qr \tag{7}$$

2

involves the $I_T^{00}(Q)$ component:

$$I_T^{00}(Q) = \frac{-i}{\pi^2} \int dk\, k^3 \int_{-1}^1 dz\, \frac{(1-z^2)}{4k^2 z^2 - Q^2} (n_+ + n_-) \tag{8}$$

with $n_\pm(k^0 \equiv \omega, T) = (e^{\omega/T \mp \mu/T} + 1)^{-1}$, μ being the chemical potential of the neutrinos.

We change now the order of the integrations and perform first the integration over Q, followed by the angular integration, that is the integration over z. The first step gives,

$$V_T(r) = -\frac{G_F^2 g_V g_V'}{2\pi^3 r} \int_0^\infty dk\, k^3\, (n_+ + n_-) \int_{-1}^1 dz\, (1 - z^2)\, \cos 2kzr. \tag{9}$$

The result of the z-integration can be cast in the form

$$V_T(r) = -\frac{G_F^2 g_V g_V'}{4\pi^3 r^4} \left[1 - r\frac{d}{dr}\right] I_T(r;\mu) \tag{10}$$

with

$$I_T(r;\mu) \equiv \int_0^\infty d\omega\, \left((e^{\omega/T - \mu/T} + 1)^{-1} + (e^{\omega/T + \mu/T} + 1)^{-1}\right) \sin 2\omega r. \tag{11}$$

The thermal integral $I_T(r;\mu)$ can be done by realising that the Fermi-Dirac distribution function can be written as an infinite series

$$\frac{1}{e^x + 1} = \sum_{n=1}^\infty (-1)^{n+1} e^{-n\,x} \tag{12}$$

Our potential is now an infinite series where every single integration can be easily performed. The final result is expressible in terms of the hypergeometric function $F(a, b; c; z)$. Indeed, we have

$$\begin{aligned}
I_T(r;\mu) &= \frac{1}{4r} \Big[F(1, -2irT; 1 - 2irT; -e^{-\mu/T}) + F(1, -2irT; 1 - 2irT; -e^{\mu/T}) \\
&+ F(1, 2irT; 1 + 2irT; -e^{-\mu/T}) + F(1, 2irT; 1 + 2irT; -e^{\mu/T}) \\
&- 8\pi rT\, \cos 2r\mu\, \operatorname{csch} 2\pi rT \Big],
\end{aligned} \tag{13}$$

which is to be plugged into Eq.(9).

Let us single out a few special cases. Start with nondegenerate neutrinos ($\mu = 0$). In that case the argument of the hypergeometric function is -1 and we may use the following property:

$$F(1, a; 1 + a; -1) = \frac{a}{2} [\psi(\frac{1}{2} + \frac{a}{2}) - \psi(\frac{a}{2})] \tag{14}$$

where $\psi(z)$ is the logarithmic derivative of the $\Gamma(z)$ function. Two further properties of $\psi(z)$ are of help here,

$$\psi(\frac{1}{2} + z) - \psi(\frac{1}{2} - z) = \pi\,\tan \pi z$$

$$\psi(z) - \psi(-z) = -\pi\,\cot \pi z - \frac{1}{z}. \tag{15}$$

After some straightforward algebra $I_T(r; \mu = 0)$ reads

$$I_T(r; \mu = 0) = \frac{1}{2r}[1 - 2\pi rT \operatorname{csch} 2\pi rT]. \tag{16}$$

Finally, the temperature dependent potential for nondegenerate relic neutrinos is:

$$V_T(r) = -V_0(r)[1 - \pi rT \operatorname{csch} 2\pi rT(1 + 2\pi rT \coth 2\pi rT)] \tag{17}$$

where $V_0(r)$ is the Feinberg Sucher potential (see Eq.(5)). At short distances, $r \ll 1.2\,mm$, i.e short compared to the distance scale set by the neutrino background temperature, the temperature dependent piece of the potential is neglegible. It is

$$V_T(r) \approx -\frac{14}{45}V_0(r)(\pi rT)^4. \tag{18}$$

At large distances (i.e. $rT \gg 1$), on the contrary, the temperature dependent effect exactly cancels the vacuum component,

$$V_T(r) \approx -V_0(r). \tag{19}$$

The other instance that we wish to explore now is the case of cold degenerate neutrinos $(T \simeq 0, \mu \neq 0)$. Here we use the relation

$$F(1, 0, 1; z) = 1 \tag{20}$$

and obtain

$$I_{T \simeq 0}(r; \mu) \simeq \frac{1}{r}(1 - \cos 2\mu r). \tag{21}$$

So finally, we have for the potential

$$V_{T \simeq 0}(r) \simeq -2V_0(r)[1 - \cos 2\mu r - \mu r \sin 2\mu r] \tag{22}$$

which agrees with the result given in [7] for this special limit[1]. To illustrate a general situation where we cannot make use of approximations, we plot the ratio $V_T(r)/V_0(r)$ as a function of distance for a chemical potential on the order of the neutrino background temperature (well within the bounds on the cosmic neutrino degeneracy that come from primordial nucleosynthesis [9] as well as structure formation studies [10]). This is depicted in Fig.2. The curve clearly shows the general trend: at short distances (much less than $1\,mm$) the effect of the neutrino relic background on the neutrino exchange potential is smallish and for distances about $1\,mm$ and beyond relic neutrinos tend to screen it off.

---

[1]The situation described in [7], namely a SN interior, involves only a background of neutrinos; as antineutrinos are not retained, the result then has an additional factor of 1/2.
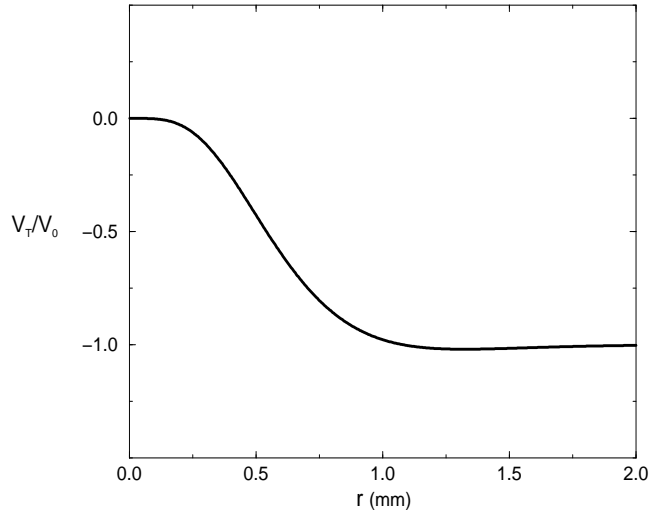
Figure 2: *Ratio between the potential V_T, at $T \sim 1.98K$ corresponding to the cosmic neutrino background, and the Feinberg-Sucher potential V_0 when $\mu/T = 1$.*

We close this paper with a brief summary. Neutrinos mediate (very feeble) long-range forces between macroscopic bodies in a vacuum. Indeed double neutrino exchange among matter fermions generates spin-independent forces that extend coherently over macroscopic distances. When the bodies lie in a neutrino heat bath these forces are altered. The phenomenon was first studied in [7] and further explored by the present authors [6]. Here we have reanalysed the effect of a neutrino background on the neutrino mediated forces using the exact Fermi-Dirac distribution function with arbitrary chemical potential. This was not done before where, for the sake of a simpler calculation, either a cold extremely degenerate (suited e.g. for supernova neutrinos) neutrino sea or a Maxwell-Boltzmann neutrino gas were used. The present analysis has led to quite different conclusions as to the long-range behaviour of the neutrino induced interactions. Indeed, the relic neutrino background, contrary to previous claims, completely cancels the long distance tail ($r \geq 1\,mm$) of the two-neutrino-exchange force and leaves the short distance ($r \ll 1\,mm$) component of the interaction unaffected. Although we still lack an experimental confirmation of the existence of relic neutrinos in spite of many suggestions to detect them [11], their theoretical status is well established within the Big Bang theory. Therefore their effect on neutrino mediated long range forces is indisputable.

References

[1] G. Feinberg, J. Sucher and C.-K. Au, Phys. Rep. **180**, 83 (1989); G. Feinberg and J. Sucher, in *Long-Range Casimir Forces: Theory and Recent Experiments in Atomic Systems*, edited by Frank S. Levin and David A. Micha (Plenum, New York, 1993).

[2] H. B. G. Casimir and P. Polder, Phys. Rev. **73**, 360 (1948); E. M. Lifschitz, JETP Lett. **2**, 73 (1956); G. Feinberg and J. Sucher, J. Chem. Phys. **48**, 3333 (1968); J. Soffer and J. Sucher. Phys. Rev. **161**, 1664 (1967); G. Feinberg and J. Sucher, Phys. Rev. **A2**, 2395 (1970); F. Ferrer and J. A. Grifols, hep-ph/9904394.

[3] G. Feinberg and J. Sucher, Phys. Rev. **A166**, 1638 (1968); S. D. H. Hsu and P. Sikivie, Phys. Rev. **D49**, 4951 (1994); J. A. Grifols, E. Masso and R., Toldra, Phys. Lett. **B389**, 363 (1996); E. Fischbach, Ann. Phys. (N.Y.) **247**, 213 (1996).

[4] V. M. Mostepanenko and I. Yu. Sokolov. Sov. J. Nucl. Phys. **46**, 685 (1987); J. A. Grifols and S. Tortosa, Phys. Lett. **B328**, 98 (1994); F. Ferrer and J. A. Grifols, Phys. Rev. **D58**, 096006 (1998); F. Ferrer and M. Nowakowski, Phys. Rev. **D59**, 075009 (1999).

[5] S. Dimopoulos, M. Dine, S. Raby and S. Thomas, Phys. Rev. Lett. **76**, 70 (1998); I. Antoniadis, S. Dimopoulos and G. Dvali, Nucl. Phys. **B516**, 70 (1998).

[6] F. Ferrer, J. A. Grifols and M. Nowakowski, Phys. Lett. **B446**, 111 (1999).

[7] C. J. Horowitz and J. Pantaleone, Phys. Lett. **B319**, 186 (1993).

[8] D. Fargion, B. Mele and A. Salis, astro-ph/9710029; T. Weiler, hep-ph/9710431; E. Waxman, astro-ph/9804023; G. Gelmini and A. Kusenko, hep-ph/9902354.

[9] H. Kang and G. Steigman, Nucl. Phys. **B372**, 494 (1992); R. V. Wagoner, W. A. Fowler and F. Hoyle, Ap. J. **148**, 3 (1967); A. Yahil and G. Beaudet, Ap. J. **206**, 26 (1976); Y. David and H. Reeves, *Physical Cosmology*, ed. by R. Balian, J. Audouze and D. N. Schramm, North-Holland, Amsterdam, 1980; J. N. Fey and C. J. Hogan, Phys. Rev. Lett. **49**, 1783 (1982); R. J. Scherrer, Mon. Not. Roy. Astron. Soc. **205**, 683 (1983). N. Terazawa and K. Sato, Ap. J. **294**, 9 (1985); K. Olive, D. N. Schramm, D. Thomas and T. Walker, Phys. Lett. **B265**, 239 (1991); G. Starkman, Phys. Rev. **D45**, 476 (1992).

[10] K. Freese, E. W. Kolb and M. S. Turner, Phys. Rev. **D27**, 1689 (1983).

[11] L. Stodolsky, Phys. Rev. Lett. **34**, 110 (1974); P. F. Smith and J. D. Lewin, Phys. Lett. **B127**, 185 (1983); I. Ferreras and I. Wasserman, Phys. Rev. **D52**, 5459 (1995); C. C. Speake and J. Leon, Class. Quantum Grav. **13**, A207 (1996); C. Hagmann, astro-ph/9905258.